Microwave Reflectometer Techniques*

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Summary—**A rigorous analysis of the microwave reflectometer is presented for what is believed to be the first time. By means of this analysis, the correct adjustment of auxiliary tuners is described, and the errors resulting from incorrect adjustments are treated in a quantitative manner.**

It is shown how the reflectometer technique may be further simplified while preserving the accuracy of measurement. A convenient method of adjusting the auxiliary tuners is described, sources of error are discussed, and an example is given of the calculation of error limits.

Introduction

The microwave reflectometer in its usual form consists of a pair of directional couplers so arranged that one couples to the forward, and the other to the reverse wave. The ratio of the sidearm outputs is, in the ideal case, equal or at least proportional to the magnitude of the reflection coefficient of the termination, from which one can calculate the standing wave ratio.

In practice, this relationship is only approximately realized because of imperfections in the directional couplers and other factors. However, directional couplers having high directivity (40 db or more) and low main guide VSWR (less than 1.05) are commercially available which permit good accuracy to be realized over a large (1.5 to 1) frequency range, while at a given frequency, further improvements may be realized by the use of auxiliary tuning.[1]

The use of auxiliary tuners has not been fully exploited or completely treated however, and in this paper a more general and rigorous analysis of the microwave reflectometer will be presented leading to the introduction of additional tuning elements. Procedures for the adjustment of these transformers will be described and a particularly simple form of the reflectometer developed. A quantitative treatment of the errors in these techniques will be presented.

General Theory

The basic form of the reflectometer is shown in Fig. 1. If b_3 and b_4 represent the voltage amplitudes of the signals at the respective detectors, the desired response is

$$\left|\frac{b_3}{b_4}\right| = |\Gamma_l| \quad \text{or} \quad \left|\frac{b_3}{b_4}\right| = K|\Gamma_l|$$

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[1] J. K. Hunton and N. L. Pappas, "The *-hp-* microwave reflectometers," *Hewlett-Packard J.*, vol. 6; September–October, 1954.

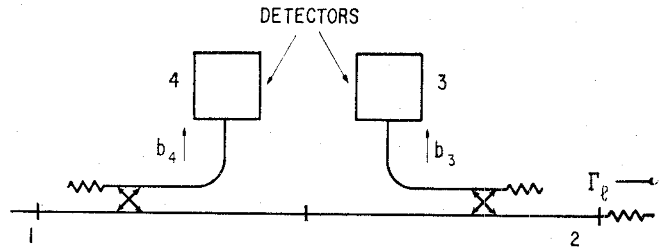

Fig. 1—Basic reflectometer.

where Γ_l is the voltage reflection coefficient of the termination on arm 2, and K is a constant whose value must be determined.

A mathematical treatment of the four arm junction and detectors shows (assuming linearity) that the response will in general be of the form:

$$\frac{b_3}{b_4} = \frac{A\Gamma_l + B}{C\Gamma_l + D} \tag{1}$$

where the A, B, C, and D are functions of the parameters of the four arm junction and detectors.

A particularly simple and convenient group of expressions for the terms A, B, C and D, may be obtained by the following procedure. The performance of the reflectometer of Fig. 1 is characterized by the functional relationships imposed upon the terminal variables a_1, b_1, a_2, and b_2, which represent the incident and emergent voltage wave amplitudes at arms 1 and 2 respectively and the responses of the detection systems (usually power) employed at arms 3 and 4. The main interest is in the relationship between the detector responses and the ratio

$$\frac{a_2}{b_2} = \Gamma_l$$

as given in (1). A variety of equivalent circuit representations may be substituted for that shown in Fig. 1 provided the relationships of interest among the terminal variables are preserved. A convenient choice for the present purpose is as follows:

Assuming that the detector impedance is constant (as will be true for example of a barretter operated at a constant resistance) mathematical models for the detectors may be constructed of lossless and matched detectors preceded by lossy fourpoles of the required parameters to produce the externally observed behavior. Reference planes in arms 3 and 4 are then chosen between these ideal detectors and the lossy discontinuities such that the latter become part of the four arm junction as shown in Fig. 2. The remainder of the system may be represented in the usual manner. This permits one to analyze the general behavior of the reflectometer as a four arm junction under the materially simplifying assumption of matched detectors on arms 3 and 4.

The main effects of this type of formulation are those of modifying the values of the scattering coefficients from those which would obtain were the reference planes in arms 3 and 4 chosen to coincide with the physical junction between the detector mounts and the associated four arm junction; and of placing the reference planes in arms 3 and 4 in a physically inaccessible position, but this is of no concern since the subsequent measurements or adjustments of the four arm junction to be described do not require access to these planes. In addition, as noted, the detector impedance is assumed to be constant. There is no further loss in generality.

An analysis of the reflectometer of Fig. 2 yields:

$$A = S_{21}S_{32} - S_{31}S_{22}$$
$$B = S_{31}$$
$$C = S_{21}S_{42} - S_{41}S_{22}$$
$$D = S_{41} \qquad (2)$$

where the $S_{m,n}$ are the scattering coefficients of the four arm junction comprised of the directional couplers and lossy fourpoles.

It is evident that the desired response will be realized if $B=C=0$. For ideal couplers of infinite directivity and a main guide VSWR of unity (and matched detectors) the terms S_{31}, S_{42}, and S_{22} are all zero, and

$$\left|\frac{b_3}{b_4}\right| = \left|\frac{A}{D}\,\Gamma_l\right|$$

as required.

In practice, the failure of the directional couplers and detectors to meet these criteria may be compensated or corrected for by the introduction of tuners at positions X and Y as shown in Fig. 3 (the tuner at Z serves an auxiliary role to be described later). The adjustment of T_X and T_Y to produce the conditions $B=C=0$ may be carried out in a variety of ways, two of which will be described. The first of these tuning procedures perhaps gives one a better feel for the conditions under which the adjustments may be physically realized, and is included for the sake of completeness, while the second method, the one which is recommended, is more convenient and potentially more accurate.

Adjustment of Tuners

In the first method, arm 2 is terminated in a matched load and T_X adjusted for a null in arm 3 (with arm 1 connected to the generator). This produces the condition $S_{31}=0$ (by definition of the scattering coefficient). The generator is then connected to arm 2; a load (not necessarily matched) connected to arm 1, and T_Z adjusted to produce a null in arm 4. If T_Y is now adjusted such that the reflection coefficient

$$\Gamma_{2i} = \frac{b_2}{a_2}$$

observed at arm 2 vanishes, it can be shown[2] that

$$S_{21}S_{42} - S_{41}S_{22} = 0,$$

and the desired operating conditions have been realized.[3]

It will be noted[2] that the first adjustment (of T_X) is independent of the second, while the converse is not true, thus the adjustments should be made in the order indicated. In addition, while T_Z is employed in the second tuning operation, once the proper adjustment of T_Y has been realized, T_Z may be readjusted (as will subsequently prove desirable) without affecting this result obtained by adjustment of T_Y.[4]

The alternative procedure will now be described. Referring again to (1), it will be evident that even before the introduction of tuning transformers, the terms B and C (for a reflectometer assembled from commercial

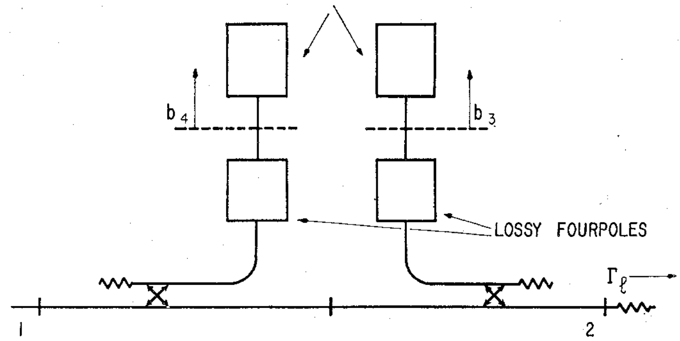

Fig. 2—Equivalent reflectometer.

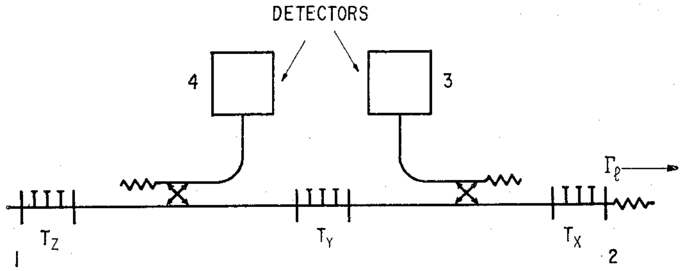

Fig. 3—Reflectometer with auxiliary tuners.

[2] The proof of this statement and several of those to follow may be effected by solving the scattering equations for the indicated quantities under the stated conditions, or by obtaining the scattering parameters of the individual components, *i.e.*, tuners and directional couplers. The proofs are generally straightforward but tedious.

[3] The minimum requirements which must be satisfied by the two couplers in order that these adjustments may be physically realized have not been determined at the time of this writing, but would appear to be well satisfied by commercially available components.

[4] The expression $(S_{21}S_{42} - S_{41}S_{22})$ may also be written

$$S_{41}\left(\frac{S_{21}S_{42}}{S_{41}} - S_{22}\right).$$

It can be shown (see note 2) that the second factor is invariant to the adjustment of tuner T_Z.

components) are quite small with regard to A and D respectively. Consider the system response to a phasable or sliding load of such magnitude that $|A\Gamma_l| \approx |B|$. From inspection it is evident that the numerator of (1) will undergo marked changes in amplitude, while the denominator remains relatively constant as the phase of the load is varied; and if T_X is adjusted to minimize the variation in the ratio

$$\left|\frac{b_3}{b_4}\right|$$

as the phase of Γ_l is varied, the condition $S_{31}=0$ will be approximately realized. The sliding load of low VSWR is then replaced by one of large VSWR (a sliding short). Variations in output as the position of the short is changed will now be predominantly due to variations in the denominator, and if T_Y is adjusted such that $|b_3/b_4|$ is again constant, the condition $S_{21}S_{42}-S_{41}S_{22}=0$ will also be very nearly realized.

A more complete mathematical treatment of the above procedure yields three solutions for a constant magnitude of b_3/b_4 as the phase of Γ_l is varied. The first solution, $A/C=B/D$ is trivial since it gives for b_3/b_4 a value which is entirely independent of Γ_l. The second solution, the one of interest, is:

$$\frac{B}{A} = \left(\frac{C}{D}\right)^* |\Gamma_l|^2 \tag{3}$$

where (*) denotes the complex conjugate.

In practice, A and D are nominally of the same order of magnitude so when (3) is satisfied, $|B|$ is of the order of $|C\Gamma_l^2|$. Thus if the sliding load has a VSWR of 1.02 ($|\Gamma_l|=0.01$), it is evident that the first tuning operation will make $|B|$ smaller than $|C|$ by a factor of approximately 10^4, while the second operation ($|\Gamma_l|\approx 1$) will reduce $|C|$ to the nominal size of $|B|$. Thus a series of these operations rapidly converges to the desired conditions $B=C=0$.

The third solution referred to above is the limiting one of a matched load ($\Gamma_l=0$) which also yields a constant ratio of b_3/b_4. The ideal or perfect match is, of course, never achieved in practice, while the closest approach to this ideal is usually by means of a variable sliding load which is adjusted to produce the minimum change of signal in an appropriate associated measuring system as the position of the load is varied. If a matched load is available, it is only necessary to adjust T_X until the output at arm 3 vanishes, while if an adjustable sliding load is used, it is convenient to carry out the operations as required to make Γ_l and b^3 vanish simultaneously. The technique described earlier, however, does not require a matched (reflection free) sliding load, but only that its reflection coefficient be small.

If the initial conditions are such that $|B| \gg |A\Gamma_l|$, the variation in the ratio

$$\left|\frac{b_3}{b_4}\right|$$

would be quite small, so as a first step (where only the phase of Γ_l is adjustable), it is usually desirable to first adjust for a null in arm 3 for an arbitrary position of the load and then adjust for a constant ratio

$$\left|\frac{b_3}{b_4}\right|$$

as the phase is varied. It will be noted that the magnitude of b_3/b_4 depends upon the magnitude of Γ_l.

It is thus of interest to note that the technique requires neither a perfectly matched load or ideal short, but only two phasable loads of different reflection coefficient magnitudes. The more closely these ideals are realized however, the more rapidly will a series of these operations converge to the desired conditions, which in practice can usually be realized to the required accuracy by only one adjustment each of T_X and T_Y. It is also of interest to note that if a perfectly matched load ($\Gamma_l=0$), an ideal sliding short ($\Gamma_l=e^{j\theta}$), and a dissipation free transformer at X are assumed, the two tuning operations, in this method, are completely independent of one another. That is, the adjustment of T_X to yield the condition $B=0$, as noted earlier, is independent of the adjustment of T_Y, while the adjustment of T_Y for the condition $B/A=(C/D)^*$ is independent of T_X.[5]

Having completed these adjustments, the response becomes:

$$\left|\frac{b_3}{b_4}\right| = \left|\frac{A}{D}\Gamma_l\right|.$$

The magnitude of the ratio A/D may be conveniently determined by observing the response to a load of known reflection, a convenient example being a fixed short for which Γ_l has the nominal magnitude 1. Thus if the response to the short is

$$\left|\frac{b_3}{b_4}\right|_s,$$

one has for the unknown reflection coefficient Γ_u.

$$|\Gamma_u| = \frac{\left|\frac{b_3}{b_4}\right|_u}{\left|\frac{b_3}{b_4}\right|_s}. \tag{4}$$

Once the reflection coefficient has been determined, the VSWR may, of course, be obtained by the usual formula

$$\sigma_u = \frac{1+|\Gamma_u|}{1-|\Gamma_u|}.$$

[5] A formal proof of this statement is somewhat lengthy, but may be recognized intuitively in the following way. An ideal sliding short preceded by a dissipation free transformer still appears as a load of $|\Gamma_l|=1$ and variable phase angle, thus the condition $B/A=(C/D)^*$ is invariant to the addition (or removal) of a lossless tuning transformer at arm 2.

A variety of techniques are available for measuring the ratio

$$\left|\frac{b_3}{b_4}\right|.$$

In general, both b_3 and b_4 will change with generator output and load impedance, requiring a ratio type meter such that

$$\left|\frac{b_3}{b_4}\right|$$

is indicated directly, or a pair of instruments to determine $|b_3|$ and $|b_4|$ individually. Alternatively, a feedback servo loop may be employed to keep $|b_4|$ constant, requiring only the observations of the values of $|b_3|$.

The functional dependence of b_4 upon the load impedance may be substantially eliminated by adjusting transformer T_Z such that the value of $|b_4|$ is independent of a sliding short at arm 2. This will reduce the amount of correction required of the servo loop, if one is employed, or the signal b_4 might be then applied to the automatic gain control channel if a standing wave amplifier were employed to measure $|b_3|$. These are only several of a number of possibilities.

If the adjustment to make $|b_4|$ independent of the load impedance has been made with sufficient care, $|b_4|$ will depend only upon the generator level, and if the generator is sufficiently stable, the signal $|b_4|$ will be constant and thus no longer contain any useful information. This first coupler with its associated detector and transformer T_Z may then be eliminated, resulting in a simplified system. This particular case appears to be enough of interest to warrant separate treatment.

A Modified Form of Reflectometer

Referring to the three arm junction of Fig. 4, the wave amplitude b_3 incident upon the detector in arm 3 can be written:

$$b_3 = b_g \frac{E\Gamma_l + F}{G\Gamma_l + H}, \tag{5}$$

where

$$E = \begin{vmatrix} S_{21} & S_{22} \\ S_{31} & S_{32} \end{vmatrix},$$

$$F = S_{31},$$

$$G = -\begin{vmatrix} -(1 - S_{11}\Gamma_g) & S_{12} & S_{13}\Gamma_d \\ S_{21}\Gamma_g & S_{22} & S_{23}\Gamma_d \\ S_{31}\Gamma_g & S_{32} & -(1 - S_{33}\Gamma_d) \end{vmatrix},$$

$$H = \begin{vmatrix} (1 - S_{11}\Gamma_g) & S_{13}\Gamma_d \\ S_{31}\Gamma_g & (1 - S_{33}\Gamma_d) \end{vmatrix},$$

b_g is the equivalent generator voltage wave amplitude, and Γ_g and Γ_d are the generator and detector reflection coefficients at reference planes 1 and 3 respectively. It will be noted that the reference planes have been chosen to coincide with the terminal surfaces of the directional coupler, thus exhibiting the dependence on the detector impedance explicitly.

Eq. (5) is of the same form as (1) and the adjustment of T_X' and T_Y' to make F and G vanish may be carried out in the manner already described except that in step two of the first method $\Gamma_{2i} = b_2/a_2$ is made to vanish by adjustment of T_Y' with arm 1 terminated in the generator impedance, instead of the procedure described earlier.

Measurement of the ratio

$$r = \frac{|b_3|_u}{|b_3|_s} = \frac{|\Gamma_u|}{|\Gamma_s|}$$

and a knowledge of $|\Gamma_s|$ again permits the calculation of $|\Gamma_u|$ or σ_u and again, a variety of techniques such as power, audio, or heterodyne detection may be employed, or a calibrated standard attenuator may be placed in the system and the changes in attenuation required to keep $|b_3|$ constant observed.

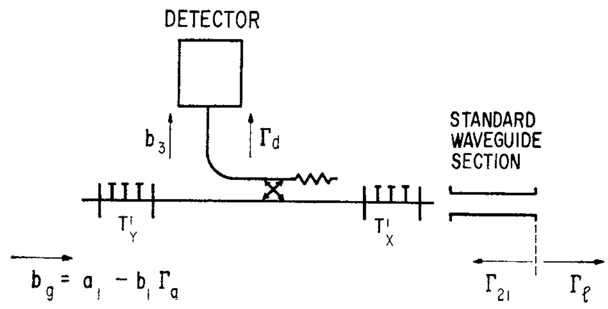

Fig. 4—Modified reflectometer.

The modified technique thus provides a reduction in the complexity of both the waveguide plumbing and associated detection equipment, but requires a signal source of stable amplitude, while the more conventional system is ideally independent of the generator level. Both systems are adaptable to either rectangular waveguide or coaxial systems, and may usually be assembled from commercially available components although it is desirable to use a precision waveguide section to terminate arm 2 in which the sliding loads may be inserted, as shown in Fig. 4. The importance of good flanges or connectors at arm 2 should also be recognized.

Analysis of Errors[6]

The sources of error, in determining $|\Gamma_u|$ and σ_u by these methods to be discussed in this section include: a) incorrect measurement of r, and uncertainty in value of $|\Gamma_s|$, and b) improper adjustment of the tuners, such that B and C do not vanish.

The consideration of other sources of error is outside the scope of this present paper, but will be treated more

[6] The analysis also applies to the modified reflectometer if the quantities E, F, G, H are substituted for A, B, C, D.

fully in a subsequent paper (in preparation) on the application of the technique to measurement of bolometer mount efficiency.

The error due to a) may be determined by inspection. If the equation for $|\Gamma_u|$ is written in the form:

$$|\Gamma_u| = |\Gamma_s| \frac{\left|\frac{b_3}{b_4}\right|_u}{\left|\frac{b_3}{b_4}\right|_s} = |\Gamma_s| r, \tag{6}$$

it is evident that the fractional error in $|\Gamma_u|$ will equal the sum of the fractional errors in $|\Gamma_s|$ and r if the latter are small.

With regard to the second item b) it will be recalled that the condition for constant output as the phase of the load is varied is:

$$\frac{B}{A} = \left(\frac{C}{D}\right)^* |\Gamma_l|^2,$$

which establishes a theoretical upper limit to the accuracy with which a particular tuning adjustment may be made.

In practice, however, the limitation usually stems from improper adjustment of the tuning transformers such that the output variations are not completely eliminated. In the discussion to follow it will be assumed that such is the case, that is, it is assumed that B and C have been reduced to the point where the variations in the expression

$$\left|\frac{b_3}{b_4}\right| = \frac{|A\Gamma_l + B|}{|C\Gamma_l + D|}$$

are due entirely to variations in the numerator or denominator as the loads of small and large VSWR are employed respectively.

A first order correction to (6) may be written as follows:

$$|\Gamma_u| = |\Gamma_s| \frac{|b_3|_u}{|b_3|_s} \left| 1 + \frac{B}{A} \frac{\Gamma_s - \Gamma_u}{\Gamma_s\Gamma_u} + \frac{C}{D}(\Gamma_s - \Gamma_u) + \cdots \right|. \tag{7}$$

The ratios $|B/A|$ and $|C/D|$ may be determined from the expressions:

$$K_1 = 20 \log \left[1 + 2\left|\frac{B}{A\Gamma_l}\right|\right]$$

and

$$K_2 = 20 \log \left[1 + 2\left|\frac{C}{D}\right|\right]$$

where K_1 and K_2 are the ratios in decibels of the maximum to minimum outputs with the sliding loads of small and large VSWR respectively, $|\Gamma_l|$ is the reflection coefficient of the load of small VSWR, and a reflection coefficient of unity has been assumed for the load of large VSWR.

Except for the presence of the factor Γ_l, these equations for K_1 and K_2 are of the same form, and values for $|B/A|$ and $|C/D|$ may be obtained from Fig. 5 where the value of $|C/D|$ is taken from the line $|\Gamma_l| = 1$. It will be noted that the evaluation of the right hand side of (7) further presupposes a knowledge of Γ_u but for the present purpose of assigning a limit of error, an accurate value is not required.

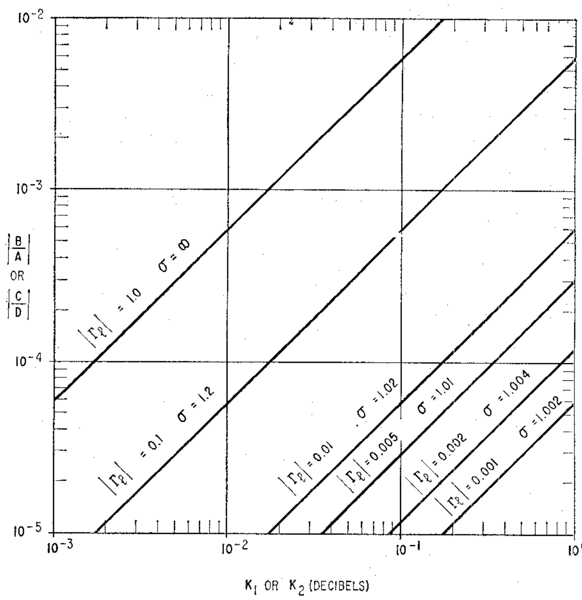

Fig. 5—Graph for the determination of $|B/A|$ and $|C/D|$.

As an example, if a sliding load of VSWR $= 1.005$ is employed in the first tuning operation, and the variation in output is reduced to 1 db, $|B/A|$ will have a value of approximately 1.6×10^{-4}. If the variation in the second step with the sliding short is reduced to 0.02 db, the value for $|C/D|$ will be approximately 1.2×10^{-3}. Assuming that the unknown load has a value $|\Gamma_u| \approx 0.2$, $(\sigma \approx 1.5)$, $|\Gamma_s| = 1.000$ (corresponding to a short-circuit) and assuming the terms in the right hand factor of (7) combine in the worst phase, values of 6 and 1.2 for

$$\left|\frac{\Gamma_s - \Gamma_u}{\Gamma_s\Gamma_u}\right|$$

and $|\Gamma_s - \Gamma_u|$ obtain respectively, for a total error of $\pm(1.6 \times 10^{-4} \times 6 + 1.2 \times 10^{-3} \times 1.2) \approx \pm 2.4 \times 10^{-3}$, or ± 0.24 per cent.

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